Equations

Usage of Equations

In Modelica equations are used for many tasks

- The main usage of equations is to represent relations in mathematical models.
- *Assignment statements* in conventional languages are usually represented as equations in Modelica
- *Attribute assignments* are represented as equations
- Connections between objects generate equations

Equation Categories

Equations in Modelica can informally be classified into three different categories

- *Normal equations* (e.g., *expr1* = *expr2*) occurring in equation sections, including `connect` equations and other equation types of special syntactic form
- *Declaration equations*, (e.g., Real x = 2.0) which are part of variable, parameter, or constant declarations
- *Modifier equations*, (e.g. x(unit="V"))which are commonly used to modify attributes of classes.

Constraining Rules for Equations

Single Assignment Rule

The total number of “equations” is identical to the total number of “unknown” variables to be solved for

Synchronous Data Flow Principle

- All variables keep their actual values until these values are explicitly changed
- At every point in time, during “continuous integration” and at event instants, the *active* equations express relations between variables which have to be fulfilled *concurrently*
  Equations are not active if the corresponding `if`-branch or `when`-equation in which the equation is present is not active because the corresponding branch condition currently evaluates to `false`
- Computation and communication at an event instant does not take time

Declaration Equations

Declaration equations:

```
constant Integer one = 1;
parameter Real mass = 22.5;
```

It is also possible to specify a declaration equation for a normal non-constant variable:

```
Real speed = 72.4;
```

declaration equations

```
model MoonLanding
  parameter Real force1 = 36350;
  parameter Real force2 = 1308;
  parameter Real thrustEndTime = 210;
  parameter Real thrustDecreaseTime = 43.2;
  Rocket          apollo(name="apollo13", mass(start=1038.358) );
  CelestialBody   moon(mass=7.382e22,radius=1.738e6,name="moon");
equation
  apollo.thrust = if (time<thrustDecreaseTime) then force1
                  else if (time<thrustEndTime) then force2
                  else 0;
  apollo.gravity=moon.g*moon.mass/(apollo.altitude+moon.radius)^2;
end Landing;
```

Modifier Equations

Modifier equations occur for example in a variable declaration when there is a need to modify the default value of an attribute of the variable

A common usage is modifier equations for the start attribute of variables

```
Real speed(start=72.4);
```

Modifier equations also occur in type definitions:

```
type Voltage = Real(unit="V", min=-220.0, max=220.0);
```

modifier equations

```
model MoonLanding
  parameter Real force1 = 36350;
  parameter Real force2 = 1308;
  parameter Real thrustEndTime = 210;
  parameter Real thrustDecreaseTime = 43.2;
  Rocket          apollo(name="apollo13", mass(start=1038.358) );
  CelestialBody   moon(mass=7.382e22,radius=1.738e6,name="moon");
equation
  apollo.thrust = if (time<thrustDecreaseTime) then force1
                  else if (time<thrustEndTime) then force2
                  else 0;
  apollo.gravity=moon.g*moon.mass/(apollo.altitude+moon.radius)^2;
end Landing;
```

Kinds of Normal Equations in Equation Sections

Kinds of equations that can be present in equation sections:

- equality equations
- `connect` equations
- `assert` and `terminate`
- `reinit`
- repetitive equation structures with `for`-equations
- conditional equations with `if`-equations
- conditional equations with `when`-equations

```
model MoonLanding
  parameter Real force1 = 36350;
  parameter Real force2 = 1308;
  parameter Real thrustEndTime = 210;
  parameter Real thrustDecreaseTime = 43.2;
  Rocket         apollo(name="apollo13", mass(start=1038.358) );
  CelestialBody  moon(mass=7.382e22,radius=1.738e6,name="moon");
equation
  if (time<thrustDecreaseTime) then
    apollo.thrust = force1;
  elseif (time<thrustEndTime) then
    apollo.thrust = force2;
  else
    apollo.thrust = 0;
  end if;
  apollo.gravity=moon.g*moon.mass/(apollo.altitude+moon.radius)^2;
end Landing;
```

conditional if-equation

equality equation

Equality Equations

```
expr1 = expr2:
(out1, out2, out3,...) = function_name(in_expr1, in_expr2, ...);
```

simple equality equation

```
class EqualityEquations
  Real x,y,z;
equation
  (x, y, z)          = f(1.0, 2.0); // Correct!
  (x+1, 3.0, z/y)  = f(1.0, 2.0); // Illegal!
                                  // Not a list of variables
                                  // on the left-hand side
end EqualityEquations;
```

Repetitive Equations

The syntactic form of a `for`-equation is as follows:

```
for <iteration-variable> in <iteration-set-expression> loop
    <equation1>
    <equation2>
    ...
 end for;
```

Consider the following simple example with a `for-`equation:

```
class FiveEquations
  Real[5]  x;
equation
  for i in 1:5 loop
    x[i] = i+1;
  end for;
end FiveEquations;
```

Both classes have equivalent behavior!

```
class FiveEquationsUnrolled
  Real[5]  x;
equation
  x[1] = 2;
  x[2] = 3;
  x[3] = 4;
  x[4] = 5;
  x[5] = 6;
end FiveEquationsUnrolled;
```

In the class on the right the `for`-equation has been unrolled into five simple equations

---

`connect`-equations

In Modelica `connect`-equations are used to establish connections between components via connectors

```
connect(connector1,connector2)
```

Repetitive `connect`-equations

```
class RegComponent
  Component components[n];
equation
  for i in 1:n-1 loop
    connect(components[i].outlet,components[i+1].inlet);
  end for;
end RegComponent;
```

Conditional Equations: `if`-equations

if *<condition>* **then**
  <equations>
elseif *<condition>* **then**
  <equations>
else
  <equations>
end if;

if-equations for which the conditions have higher variability than constant or parameter must include an *else-part*

Each `then`-, `elseif`-, and `else`-branch must have the *same number of equations*

```
model MoonLanding
  parameter Real force1 = 36350;
  ...
  Rocket        apollo(name="apollo13", mass(start=1038.358) );
  CelestialBody moon(mass=7.382e22,radius=1.738e6,name="moon");
equation
  if (time<thrustDecreaseTime) then
    apollo.thrust = force1;
  elseif (time<thrustEndTime) then
    apollo.thrust = force2;
  else
    apollo.thrust = 0;
  end if;
  apollo.gravity=moon.g*moon.mass/(apollo.altitude+moon.radius)^2;
end Landing;
```

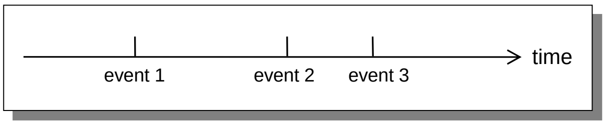
Conditional Equations: `when`-equations

when *<conditions>* **then**
  <equations>
end when;

```
when x > 2 then
  y1 = sin(x);
  y3 = 2*x + y1+y2;
end when;
```

<*equations*> in when-equations are instantaneous equations that are active at events when <*conditions*> become true

Events are ordered in time and form an event history:



- An event is a *point* in time that is instantaneous, i.e., has zero duration
- An *event condition* switches from false to true in order for the event to take place

Conditional Equations: `when`-equations cont'

when *<conditions>* **then**
  <equations>
end when;

when-equations are used to express instantaneous equations that are only valid (become active) *at events*, e.g. at discontinuities or when certain conditions become true

```
when x > 2 then
  y1 = sin(x);
  y3 = 2*x + y1+y2;
end when;
```

```
when {x > 2, sample(0,2), x < 5} then
  y1 = sin(x);
  y3 = 2*x + y1+y2;
end when;
```

```
when initial() then
  ...   // Equations to be activated at the beginning of a simulation
end when;
...
when terminal() then
  ...   // Equations to be activated at the end of a simulation
end when;
```

Restrictions on `when`-equations

Form restriction

```
model WhenNotValid
  Real x, y;
equation
  x + y = 5;
  when sample(0,2) then
    2*x + y = 7;
    // Error: not valid Modelica
  end when;
end WhenNotValid;
```

Modelica restricts the allowed equations within a `when`-equation to: variable = expression, if-equations, for-equations,...

In the `WhenNotValid` model when the equations within the `when`-equation are not active it is not clear which variable, either x or y, that is a “result” from the `when`-equation to keep constant outside the when-equation.

A corrected version appears in the class `WhenValidResult` below

```
model WhenValidResult
  Real x,y;
equation
  x + y = 5;            // Equation to be used to compute x.
  when sample(0,2) then
    y = 7 - 2*x;        // Correct, y is a result variable from the when!
  end when;
end WhenValidResult;
```

Restrictions on when-equations cont’

Restriction on nested `when`-equations

```
model ErrorNestedWhen
  Real x,y1,y2;
equation
  when x > 2 then
    when y1 > 3 then    // Error!
      y2 = sin(x);      // when-equations
    end when;           // should not be nested
  end when;
end ErrorNestedWhen;
```

`when`-equations cannot be nested!

Restrictions on when-equations cont’

Single assignment rule: same variable may not be defined in several `when`-equations.

A conflict between the equations will occur if both conditions would become true at the same time instant

```
model DoubleWhenConflict
  Boolean close;    // Error: close defined by two equations!
equation
  ...
  when condition1 then
    close = true;   // First equation
   end when;
  ...
  when condition2 then
    close = false;  //Second equation
  end when;
end DoubleWhenConflict
```

Restrictions on `when`-equations cont’

Solution to assignment conflict between equations in independent `when`-equations:

- Use `elsewhen` to give higher priority to the first when-equation

```
model DoubleWhenConflictResolved
  Boolean close;
equation
  ...
  when condition1 then
    close = true;  // First equation has higher priority!
  elsewhen condition2 then
    close = false; //Second equation
  end when;
end DoubleWhenConflictResolved
```

Restrictions on `when`-equations cont’

Vector expressions

The equations within a `when`-equation are activated when any of the elements of the vector expression becomes true

```
model VectorWhen
  Boolean close;
equation
  ...
  when {condition1,condition2} then
    close = true;
  end when;
end DoubleWhenConflict
```

`assert`-equations

assert(assert-expression, message-string)

`assert` is a predefined function for giving error messages taking a Boolean condition and a string as an argument

The intention behind `assert` is to provide a convenient means for specifying checks on model validity within a model

```
class AssertTest
  parameter Real lowlimit = -5;
  parameter Real highlimit = 5;
  Real x;
equation
  assert(x >= lowlimit and x <= highlimit,
         "Variable x out of limit");
end AssertTest;
```

`terminate`-equations

The `terminate`-equation successfully terminates the current simulation, i.e. no error condition is indicated

```
model MoonLanding
  parameter Real force1 = 36350;
  parameter Real force2 = 1308;
  parameter Real thrustEndTime = 210;
  parameter Real thrustDecreaseTime = 43.2;
  Rocket    apollo(name="apollo13", mass(start=1038.358) );
  CelestialBody moon(mass=7.382e22,radius=1.738e6,name="moon");
equation
  apollo.thrust = if (time<thrustDecreaseTime) then force1
                  else if (time<thrustEndTime) then force2
                  else 0;
  apollo.gravity = moon.g * moon.mass /(apollo.height + moon.radius)^2;
  when apollo.height < 0 then      // termination condition
    terminate("The moon lander touches the ground of the moon");
  end when;
end MoonLanding;
```